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The Adaptive Synchronization of fractional-order Economic chaotic system

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Abstract— In this paper, the chaos control and the synchronization of two fractional-order Economic chaotic systems studied. According to the Lyapunov stabilization theory and the adaptive control theorem, the adaptive control rule is obtained for described error dynamic stabilization. Finally, the numerical simulation illustrates the efficiency of the proposed method in synchronizing two chaotic systems

Index Terms—. Fractional Derivative systems, Adaptive control, Chaotic System

1 INTRODUCTION

The chaotic behavior of dynamic systems can be observed in several real applications in the world, such as circuits, mathematics, power systems, medicine, electrochemical biology, etc. [1, 2]. Thus, chaos is one of the most interesting subjects to attract the experts from various fields of study. The fractional calculus established about 300 years ago. But, its applications in physics and engineering have been studied just in recent decades. Most of the practical and industrial systems could be modeled with fractional-order derivations [14]. Recently, the control and synchronization of chaotic systems is one of the most appealing subjects, which have attracted many scientists [15]. For instance, in [3], the stabilization of an integrated fractional-order chaos system is studied. In [4], the stabilization of the fractional-order system by using the active control method is investigated. In [5], a method for the stabilization of a fractional-order system based on the active sliding mode is presented. In [6], the Routh-Horvitz method in fractional-order systems is used to synchronize the Duffing-vander Pol fractional-order chaos system. In [7], a smart resistant fractional-order sliding level is determined and the sliding control is studied for a non-linear system. In [8], a new hyper chaotic fractional-order system is presented and the synchronization of a class of non-linear fractional-order systems is considered. In [9], the adaptive sliding mode control for a new class of chaotic fractional-order systems, which are non-deterministic are proposed. For this aim, the fractional-order derivation is used to produce sliding level. In [10], the authors analyze the behavior of fractional-order chaos systems, investigating the stabilization conditions by using the projective method. In [11], a simple but efficient way to control the fractional-order chaos system, using the TS fuzzy model and adaptive regulation mechanism is presented. In [12], the second-order sliding mode control to stabilize one class of non-deterministic fractional-order system with external disturbances is studied. In [13], an adaptive fractional-order feedback controller to stabilize the chaos systems is presented. Then, a simple but practical method to synchronize the fractional-order chaos system is investigated.

In this paper, a fractional-order chaos system with unknown parameter is considered, for which the adaptive controller is designed. Using the Liapanov theory and the appropriate adaptive rule, the control rule is proved. The organization of this paper is as follows: In part II the basic concepts of calculus is presented. In section III, the problem is introduced. Section IV includes the process of obtaining the adaptive control and the parameter estimation rule to synchronize the fractional-order chaos system with unknown parameters. In Section V, the simulation results of the proposed method performance are presented. Section VI contains the conclusion of appropriate performance of this method to synchronize the fractional-order chaos systems.

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MATHEMATICAL PRELIMINARIES

The derivative-integrator operator is represented by ${}_aD_t^q$, which is used to show the fractional derivation and integral operator. For the positive values of q, it is a derivation symbol and for the negative values of q it turns into an integral symbol. The definitions usually used for the fractional derivation are Grunwald–Letnikov, Riemann-Liouville and Caputo.

$$ {}_aD_t^q = D^q = \begin{cases} \dfrac{d^q}{dt^q}, & q > 0 \\ 1, & q = 0 \\ \int_a^t (d\tau)^q \triangleq I^{-q}, & q < 0 \end{cases} \quad (1) $$

The second definition of Riemann-Liouville [14] is the definition of RL, which is known as the simplest one as follows:

$$ {}_aD_t^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d}{dt} \int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau, \quad (2) $$

where, $n-1<q<n$ and $\Gamma(.)$ is the Gamma function.

THE PROBLEM DESCRIPTION

Designing an adaptive controller for the synchronization of a chaos system:

In order to synchronize the behavior of chaotic system, the Economic system with three degrees of freedom is defined by the following equations:

$$\begin{cases} D^{q1}x = z+(y-a)x \\ D^{q2}y = 1-by-x^2 \\ D^{q3}z = -x-cz \end{cases} \quad (3)$$

where, x, y, z are the state variables and $a,b,c \in R$ are the system parameters.

To synchronize two systems, The Master system is defined as follows:

$$\begin{cases} D^{q1}x_1 = z_1+(y_1-a)x_1 \\ D^{q2}y_1 = 1-by_1-{x_1}^2 \\ D^{q3}z_1 = -x_1-cz_1 \end{cases} \quad (4)$$

where, z_1, y_1, x_1 are the Master system variables and q_1, q_2, q_3 is the order of fractional derivation. The Slave system is as follows:

$$\begin{cases} D^{q1}x_2 = z_2+(y_2-a)x_2+u_1(t) \\ D^{q2}y_2 = 1-by_2-{x_2}^2+u_2(t) \\ D^{q3}z_2 = -x_2-cz_2+u_3(t) \end{cases} \quad (5)$$

where, z_2, y_2, x_2 are the variables of Slave system, q_1, q_2, q_3 represents the order of fractional derivation and the design control signal to synchronize two systems are defined by u_1, u_2, u_3.

ADAPTIVE CONTROLLER DESIGN

In this section, the synchronization of two chaotic systems with unknown parameters are studied.

Define

$$e_x = x_2 - x_1; \quad e_y = y_2 - y_1; \quad e_z = z_2 - z_1; \qquad (6)$$

By subtracting Eq. 4 from Eq. 5, the error dynamic equation is obtained as follows:

$$\begin{aligned} D^{q1}e_x &= e_z + y_2x_2 - y_1x_1 - ae_x + u_1(t) \\ D^{q2}e_y &= -be_y - x_2^2 + x_1^2 + u_2(t) \\ D^{q3}e_z &= -e_x - e_z + u_3(t) \end{aligned} \quad (7)$$

***Theorem*:** If the control rule is

$$\begin{aligned} u_1(t) &= -e_z - y_2x_2 + y_1x_1 + ae_x - k_1D^{1-q1}ex \\ u_2(t) &= be_y + x_2^2 - x_1^2 - k_2D^{1-q2}ey \\ u_3(t) &= e_x + ce_z - k_3D^{1-q3}ez \end{aligned} \quad (8)$$

where, A_1, A_2, A_3 are the positive coefficients, then the states of system in Eq. 5, are approximated asymptotically to the states of system in Eq. 4.

Proof: To prove the synchronization of two systems in Eq. 4 and Eq. 5, using the control rule equation shown in Eq. 8 , the stabilization of system must be investigated. For this aim, the desired Lyapunov function should be definite positive with negative definite derivation along the trajectory of the system.

The Lyapunov function is defined as follows:

$$V = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2\right) \quad (9)$$

By differentiation from Eq. 9, we derive:

$$\begin{aligned} \dot{V} &= e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + \dot{\tilde{a}}\tilde{a} + \dot{\tilde{b}}\tilde{b} + \dot{\tilde{c}}\tilde{c} \\ &= e_xD^{1-q1}(D^{q1}e_x) + e_yD^{1-q2}(D^{q2}e_y) + e_zD^{1-q3}(D^{q3}e_z) \end{aligned} \quad (10)$$

Substituting Eq. 7 in Eq. 10, we have

$$\begin{aligned} &= e_xD^{1-q1}\left({}_z + y_2x_2 - y_1x_1 - ae_x + u_1(t)\right) \\ &+ e_yD^{1-q2}\left(-be_y - x_2^2 + x_1^2 + u_2(t)\right) \\ &+ e_zD^{1-q3}\left(-e_x - e_z + u_3(t)\right) \end{aligned} \quad (11)$$

The control inputs u_1, u_2, u_3 must be selected, so that the values of V and $\dot{V}$ are the definite positive and negative respectively. Thus, by substituting u_1, u_2, u_3 from Eq. 8 and the parameters estimation rules in Eq. 9 into Eq. 11, the following relation is achieved:

$$\dot{V} = -A_1{e_x}^2 - A_2{e_y}^2 - A_3{e_z}^2 < 0 \quad (12)$$

Therefore, the available control rule shown in Eq. 8, the system states in Eq. 5 are asymptotically approximated by the system states of Eq. 4. In other word, resulting in the approximated zero synchronization error.

NUMERICAL SIMULATIONS

At this section, the efficiency of our proposed method is assessed. The simulation results are performed on the synchronization of Economic- fractional chaotic

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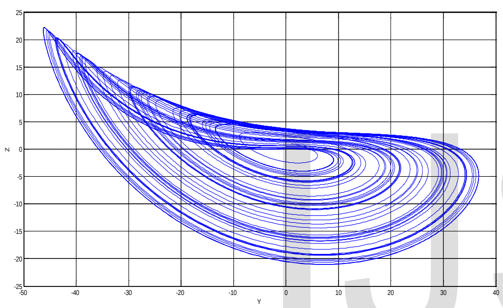
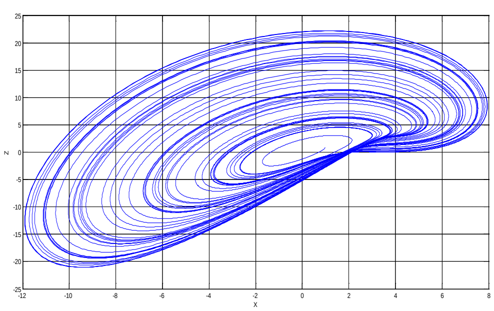
systems with different initial states. In this simulation, the sampling time and the order of fractional derivation are defined as h=0.001 and $q_1 = 0.84, q_2 = 0.82, q_3 = 0.80$ respectively. The initial states are as follows:

$$[d^{q1-1}x_1(0), d^{q2-1}y_1(0), d^{q3-1}z_1(0)]^T = [2,3,2]^T$$
$$[d^{q1-1}x_2(0), d^{q2-1}y_2(0), d^{q3-1}z_2(0)]^T = [3,4,3]^T$$

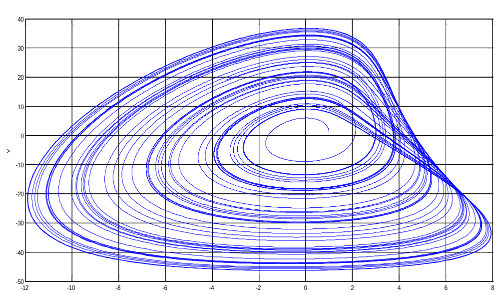
For some specific values of $a, b, c,$ the system identified with Eq. 3 is turned to be a chaotic system whose states' behavior for the values of $a = 3.2, b = 1.01, c = 1.05,$ are illustrated in Fig. 1. In addition, its chaotic behavior and butterfly effect treatment are depicted in the same figure. In Fig. 2, the synchronization performance of fractional-order chaotic systems in Eqs 4 and 5 are presented for all possible states. According to Fig. 3, it is evident that the synchronization error converges to zero. In Fig. 4, the control signal for three states is plotted.



a :The chaotic trajectory of Eq. 3 for states y, z

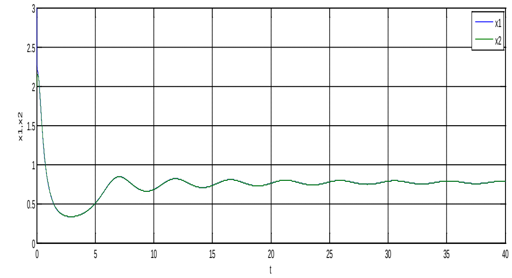


b :The chaotic trajectory of Eq. 3 for states x ,z

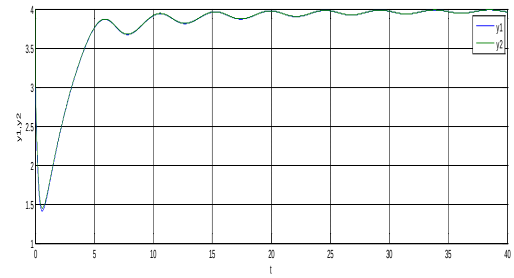


c : The chaotic trajectory of Eq. 3 for states y, x

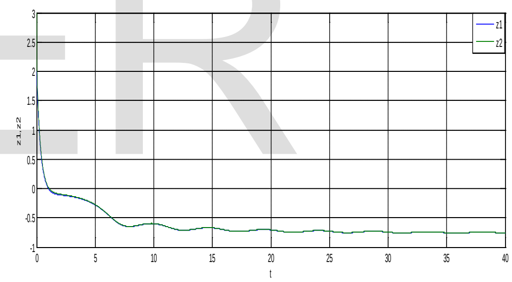
Fig. 1. (a,b,c) The chaotic trajectory of Eq. 3 for three states



a :The synchronization performance of fractional-order chaotic systems shown in Eqs. 4 and 5 for x1, x2

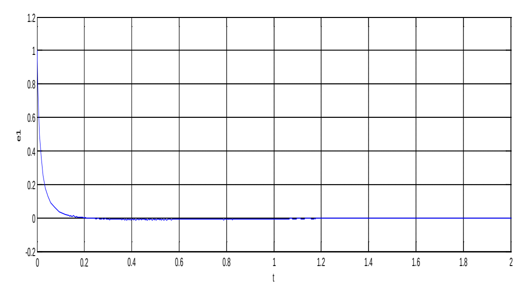


b :The synchronization performance of fractional-order chaotic systems shown in Eqs. 4 and 5 for y1,y2



c :The synchronization performance of fractional-order chaotic systems shown in Eqs. 4 and 5 for z1,z2

Fig. 2: (a,b,c):The synchronization performance of fractional-order chaotic systems in Eqs. 4 and 5 for three states



a :The synchronization error of fractional-order chaotic system for state x

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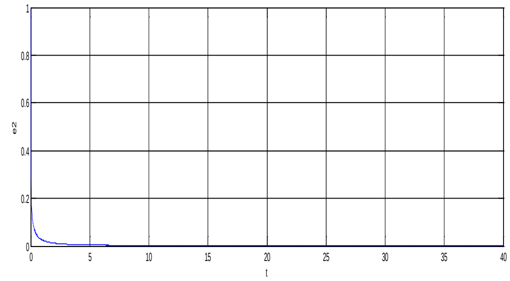
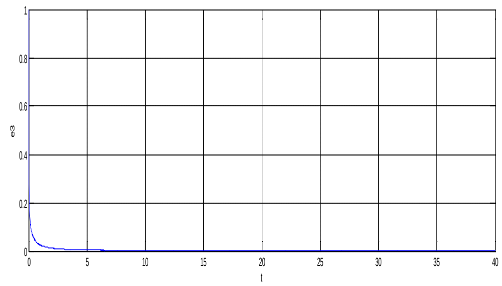
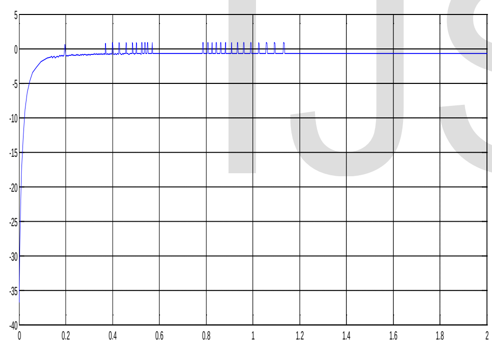


b :The synchronization error of fractional-order chaotic system for state y

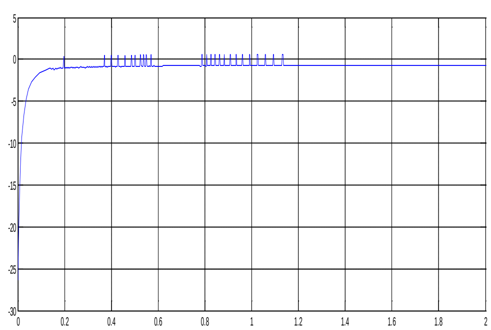


c :The synchronization error of fractional-order chaotic system for state z

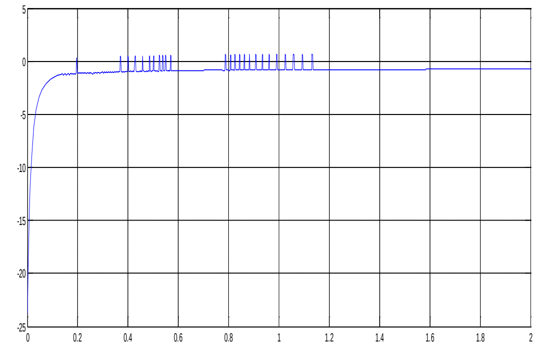
Fig. 3. (a,b,c): The synchronization error of fractional-order chaotic system for three states



a :The control signal u1



b : The control signal u2



c : The control signal u3

Fig. 4. (a,b,c) The control signal performance for three states

Conclusion

In this paper, the synchronization issue of fractional-order chaotic systems by using the adaptive control method has been investigated. According to the stabilization theory of Lyapunov and the adaptive control theory, a controller was designed to stabilize and synchronize fractional-order chaotic systems. In addition to obtain an adaptive control rule, the synchronization rule for unknown parameters of the system was also presented. Finally, the simulation results demonstrated the appropriate performance of this method in order to synchronize the fractional-order chaotic systems.

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